EXPERIMENTAL INVESTIGATION OF THE WAVE STRUCTURE OF A SUPERSONIC BOUNDARY LAYER

A. D. Kosinov, A. A. Maslov, and S. G. Shevel'kov UDC 532.526

At present there are a number of theoretical techniques to predict laminar boundary-layer transition to turbulence based on different criteria. In order to develop a general theory for transition it is necessary to have a detailed description of wave processes in the boundary layer and theoretical and experimental results on stability. The growth of natural disturbances [1] is, as a rule, studied in experimental investigations on the stability of supersonic boundary layers. The shortcoming of such studies is the absence of the complete three-dimensional characteristics of the disturbance field in the boundary layer. Hence it is possible to expect only qualitative agreement between theory and experiments. A more accuracy quantitative comparison is possible with controlled artificial disturbances modeling wave growth in the boundary layer. An alternate approach (in the study of the development of natural disturbances) consists of corresponding correlation measurements followed by an evaluation of wave spectra. Such an approach was partially realized in [2]. A point disturbance source was used in [3] to study the stability of incompressible boundary layers. The wave field created by it (amplitude and phase distribution of fluctuations in space) was recorded by hot-wire anemometry. The results of the study reflect the evolution of monochromatic plane waves in the boundary layer. A method similar to [3] was used in [4, 5] to study the stability of supersonic boundary layers. An electric arc was used as the disturbance source. It is shown that the given method makes it possible to study the growth of characteristic waves in supersonic boundary layers. Some problems that arise while obtaining experimental data and also during their analysis have been discussed in [4]. The present study extends the approach of [3, 4] and new experimental results have been obtained. Stability analysis of the boundary layer using linear approximation has been carried out on the basis of the experimental data.

Experiments were conducted in the low-turbulence supersonic wind tunnel T-325 at the Institute of Theoretical and Applied Mechanics (ITPM), Siberian Branch, Academy of Sciences of the USSR [6] with a test section 200 × 200 mm at Mach number M = 2.01 and a unit Reynolds number $Re_1 = 6.58 \cdot 10^6$/meter. A flat plate made of steel was used with a rounded leading edge swept along the side at an angle 14°30' (the leading-edge radius b ≈ 0.04 mm). The plate length was 450 mm, width 200 mm, and thickness 10 mm. The model was mounted horizontally at the midplane of the test section at zero angle of attack. The disturbance source was located inside the model. Its basic part (rotary electrode in a ceramic tube to regulate arc interval) was mounted perpendicular to the plane of the model. The disturbance was introduced into the boundary layer through an orifice in the model surface. The orifice diameter was 0.5 mm. The coordinates of the disturbance source were: x = 16.7 mm and z = 0, where x is the streamwise coordinate measured from the model leading edge and z is the transverse coordinate.

A constant-current hot-wire anemometer (TPT-3) [7] was used to determine the disturbances. Sensors made of tungsten were used with a diameter of 6 μm and a length of ~1.3 mm. The sensor was traversed along x, y, and z (y is the distance normal to the flat plate). The coordinates x and z were measured with accuracies of 0.1 mm.

The selective amplifier U2-8 was used as the frequency filter. Fluctuating amplitude in the narrow band was recorded by an rms (root-mean-square) voltmeter built into U2-8. The phase of the signal in relation to the disturbance source was determined by dual-beam oscillograph S1-17, synchronized with the hot arc.

The mean flow parameters (M, Re_1, stagnation temperature, free-stream velocity U) were determined with the data-logging system provided for the wind-tunnel facility. Error in measuring Mach number was 1% and the unit Reynolds number was measured with error not exceeding 3%.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 107-112, September-October, 1986. Original article submitted July 1, 1985.

 0021-8944/86/2705-0730$12.50

The voltage across the anemometer bridge was kept constant when the sensor was traversed along x and z (this was achieved by moving the sensor along y) which approximately corresponds to measurements along isotachs and y/δ = const, where δ is the boundary layer thickness. The overheat for the sensor was 0.8 and, as a result, it is possible to state that primarily the amplitude and phase of the fluctuations of the mass flux were fixed [2].

Measurements were made at $y/\delta \approx 0.5$-0.6 which corresponds to the maximum level of fluctuations in the boundary layer. The measurement interval along x was 5 mm, and varied from 0.2 to 1 mm along z. The measured data were analyzed using Fourier spectra (discrete Fourier transform [8]) based on the wave number β in the z direction. Amplitude distribution along z had zero values at the ends of the interval, which made it possible to go to finite limits in integration.

It is necessary to use special weighting functions, viz., spectral windows [9, 10] in order to determine Fourier spectra from discretely available functions (distribution) if only a part of the distribution is known. By using the appropriate window it is then possible to significantly reduce the influence of gaps at the ends of the function and in a number of cases estimate the spectral distribution close to the correct one.

Spectra for the wavenumber α_r along the streamwise coordinate x have been similarly obtained in the present study. As shown in [10], the most effective window for such a case is the Kaiser–Bessel window. Numerical values for the window in the present study were determined with an accuracy to the sixth decimal place. Spectral verification of the Kaiser–Bessel window characteristics gave results agreeing with [10]. The program for Fourier transform was verified for a number of test functions with and without the window.

The wave angle was determined from the relation $\chi = \arctan(\beta/\alpha_r)$, and the phase velocity of the disturbance in the x direction from the equation $c_x = 2\pi f/\alpha_r U$, where f is the disturbance frequency.

The spatial amplification factor for the Tollmien–Schlichting waves were estimated from

$$-\alpha_i(\beta) = 0{,}5\partial \ln[A(\mathrm{Re},\ \beta)]/\partial\,\mathrm{Re}.$$

Here $\mathrm{Re} = (\mathrm{Re}_1 x)^{1/2}$; $A(\mathrm{Re},\ \beta)$ is the amplitude of Tollmien–Schlichting wave.

Experimental data were analyzed to determine Fourier components of the wave packet whose evolution was compared with computations based on linear stability theory. In the parallel-flow approximation in the boundary layer, the solution to the linear problem is sought in the form

$$Q(x,\ y,\ z,\ t) = q\,(y)\exp\,(i\alpha x + i\beta z - i\alpha ct),$$

where q(y) are the disturbance amplitude functions for velocity, density, and temperature. The eigenvalues were determined by numerical integration of the system of stability equations with homogeneous boundary conditions, using Dunn–Lin approximation. Orthogonalization technique was used in the integration process. Computations were carried out for the boundary layer on an insulated flat plate at $M = 2.0$ in accordance with the experiment. The Sutherland viscosity law was assumed, the ratio of specific heats $\gamma = 1.41$, and the Prandtl number $\sigma = 0.72$. The system of equations and the method of solution are described in detail in [1]. The spatial amplification factor α_i and disturbance phase velocity C_x along the streamwise direction were obtained from numerical integration conducted for the nondimensional frequency parameter $F = 0.188\cdot10^{-4}$, $0.371\cdot10^{-4}$, and $0.547\cdot10^{-4}$ (which correspond to disturbance frequencies f = 10, 20, and 30 kHz); here, $F = 2\pi f/\mathrm{Re}_1 U$.

The stability of the boundary layer on the test model with respect to "natural" (with arc switched off) disturbances was investigated immediately before experiments with artificial disturbances. It was found that within experimental measurement errors for the disturbance amplitude (~10%), the orifice for the disturbance source in the model test surface does not cause the development of disturbances in its wake. The nature of the disturbance growth for f = 10 kHz corresponds to the region on the first branch of the neutral curve, f = 20 kHz corresponds to maximum instability, and f = 30 kHz is the region close to the second branch of the neutral stability curve for Tollmien–Schlichting waves.

The primary objective of the experimental part of the study was the measurement of the disturbance amplitudes and phases in the boundary layer at fixed frequencies. A two-dimen-

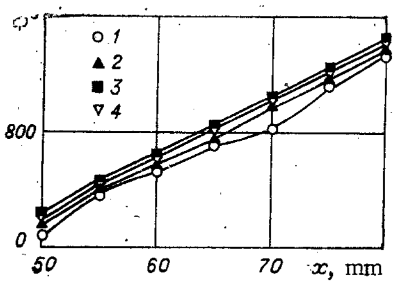

Fig. 1

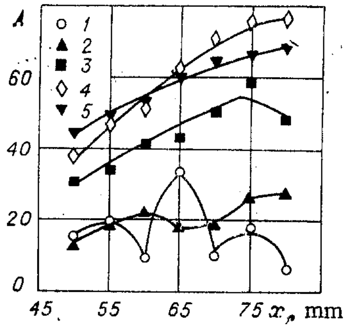

Fig. 2

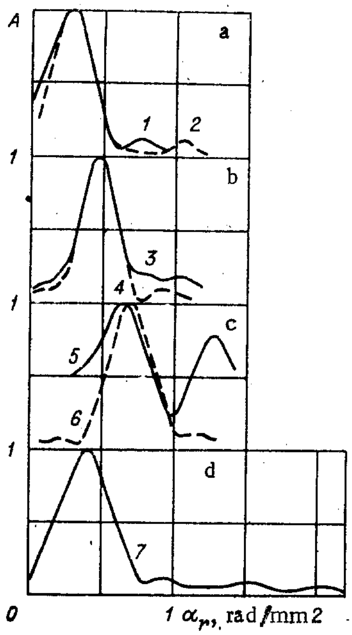

Fig. 3

sional data network has been experimentally obtained whose Fourier analysis made it possible to go back to the consideraton of plane waves in the boundary layer. A detailed description of the technique used is given in [11]. The range of measurements along x was from 50 to 80 mm for f = 10 and 30 kHz and from 50 to 90 mm for f = 20 kHz.

The variations of disturbance phase Φ and amplitude A with frequency f = 30 kHz along x are shown in Figs. 1 and 2 at fixed values of wave number β. Symbols 1-4 correspond to β = 0, 0.34, 0.68, and 0.945 rad/mm, and the symbol 5 in Fig. 2 is for β = 1.24 rad/mm. It is seen from Fig. 1 that the increase in phase is nearly linear when β = 0.68 and 0.945. The dependence of phase on x is significantly different from a linear relation for small values of β (especially for β = 0). The amplitude growth (Fig. 2) along x for β = 0 has a modulating character for the modulation weakens with increasing β. Observing all these curves together, it is possible to state that a number of waves are propagated along x and their combination results in the amplitude modulation.

The disturbance spectra for the wavenumber α_r are shown in Fig. 3, where the symbols represent the following values of β: 1, 3, 5, 7) β = 0; 2) β = 0.67; 4) β = 0.84; and 6) β = 0.58 rad/mm. Results are given for disturbances with frequencies f = 10 kHz (a), f = 20 kHz (b, c), and f = 30 kHz (d); the spectra 1, 5, and 6 are obtained from seven points, 3 and 4 from nine points, and 7 from thirty-one. The maximum peaks in all plots correspond to Tollmien–Schlichting waves and the other peaks are "acoustic" disturbances (their phase velocity $C_x \leq 0.3$). Consider the spectra 1-6. The spectral range is limited here by the interval along x: Δx = 5 mm, $\Delta\alpha_r \approx 2\pi/\Delta x \approx 1.25$ rad/mm. The width of spectral peaks is primarily determined by the duration of the realization along x (parameters of the spectral window also influence the width). It is seen that for β = 0, the "acoustic" content in the spectra increases with increases in frequency. If the transformation of disturbances from

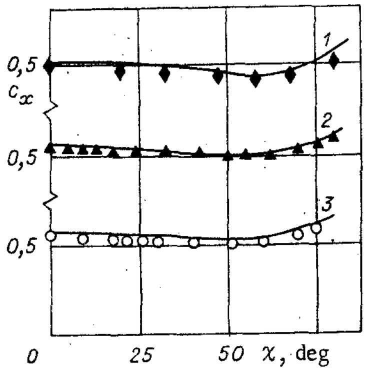

Fig. 4

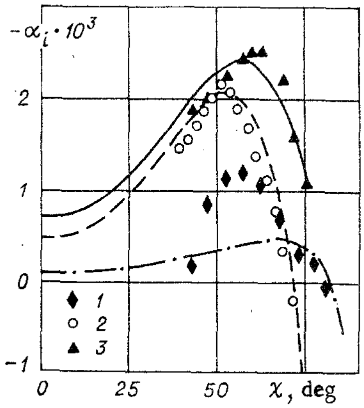

Fig. 5

the source into waves in the boundary layer takes place identically for these test frequencies, then it is possible to establish a dependence between the amplitude of sound and the nature of the growth of Tollmien–Schlichting waves (for the spectra near $\beta = 0$); near the first branch of the neutral curve the Tollmien–Schlichting wave is an order of magnitude bigger than acoustic waves ($F = 0.188 \cdot 10^{-4}$), in the maximum growth region ($F = 0.371 \cdot 10^{-4}$) it is 5-6 times larger, and near the second branch of the neutral curve ($F = 0.547 \cdot 10^{-4}$) the disturbance function in the boundary layer exceeds acoustic waves by a factor of 1.5 to 2. This means that acoustic waves grow faster than characteristic disturbances when $\beta \approx 0$. The shortcomings of the studies [12, 13] on the growth of two-dimensional Tollmien–Schlichting waves ($\beta = 0$) are thus clear. The large contribution to the total signal from acoustic disturbances for the given spread angle did not allow the observation of the evolution (as a function of Re) of Tollmien–Schlichting waves. For the spectra of α_r, where $\beta \approx$ 0.3-2 rad/mm (lines 2, 4, and 6 in Fig. 3) the relation between acoustic disturbances and waves of the characteristic type practically does not change with the frequency parameter, and that means both disturbances grow in an identical manner. These results are in agreement with results of [4], where the sound radiated by supersonic boundary layer was concentrated at $\beta = 0$, indicating two-dimensionality of the wave.

The spectral range shown in Fig. 3a-c is not sufficient for completely covering the wave spectra in the boundary layer. The problem is further complicated by the fact that the measurement region where the disturbance growth is linear is 5-6 times the wavelength of Tollmien–Schlichting waves, which puts a limit on the minimum width of spectral peaks and their solution using Fourier analysis. The amplitude spectrum for wavenumber α_r shown in Fig. 3d is based on results from 31 measurement locations along x with a more frequent step ($\Delta x = 1$ mm) which made it possible to extend the spectral range for α_r by a factor of 5 ($\Delta\alpha_r \approx 6$ rad/mm). It is possible to observe a number of peaks, almost equally spaced from each other, and the largest peak represents Tollmien–Schlichting waves and the others are acoustic disturbances radiated by the laminar boundary layer.

Thus, the above-described source excited a broadband spectrum of disturbances in the boundary layer. It is possible to state that this spectrum is discrete within experimental accuracy. Discretization along the coordinate x superimposes the limitation on the interpretation of the spectra: it is not possible to give a unique answer to the question whether or not there are waves traveling upstream. A unique answer is possible if $\Delta\alpha_r$ is such that it contains the entire spectrum. It is not possible to establish it in the present study.

The spectrum for α_r has also been evaluated for twice the number of realizations along x with a 1-mm step but without Fourier analysis for β (phase and amplitude measurements were made along the line $z = 0$ at the center of the packet along x). This evaluation is close to the spectrum in Fig. 3d but has narrower peaks and to the left of the peak corresponding to the Tollmien–Schlichting wave it has a new peak with a phase velocity $c_x \sim 0.9$-1.1. It is possibly a vortex from the source spreading along the outer edge of the boundary layer.

Since the acoustic disturbances in the spectra decrease with increase in β and for a given range of β they could be neglected in comparison with characteristic disturbances, it is possible to apply the above-described technique for a detailed investigation of the growth of three-dimensional Tollmien–Schlichting waves.

Phase velocities of characteristic disturbances are shown in Fig. 4 and the spatial growth is given in Fig. 5 (Re = 653 ± 4). The points 1-3 represent frequency parameters $F = 0.188\cdot10^{-4}$, $0.371\cdot10^{-4}$, and $0.547\cdot10^{-4}$, and the lines correspond to respective computed results. There is a good agreement between theory and experiment for phase velocities. The deviation is significant for α_r at f = 10 kHz, but it is worth noting that the computation was carried out for the boundary layers on a sharp plate, whereas a flat plate with a rounded leading edge was used in the experiment.

LITERATURE CITED

1. S. A. Gaponov and A. A. Maslov, Disturbance Growth in Compressible Flow [in Russian], Nauka, Novosibirsk (1980).
2. J. M. Kendall, "Wind-tunnel experiments relating to supersonic and hypersonic boundary-layer transition," AIAA J., 13, No. 3, 290 (1975).
3. V. M. Gilev, Yu. S. Kachanov, and V. V. Kozlov, Spatial Growth of Wave Packet in Boundary Layer [in Russian], Preprint No. 34, Inst. for Theoretical and Applied Mechanics (ITPM), Siberian Branch, Academy of Sciences of the USSR (1981); Izv. Sib. Otd. Akad. Nauk SSSR No. 13 (1983); Ser. Tekh. Nauk, Vyp. 3.
4. A. D. Kosinov, A. A. Maslov, and N. V. Semenov, Techniques for the Introduction of Spatial Wave Packet in Boundary Layer [in Russian], Preprint No. 34, Inst. of Theoretical and Applied Mechanics (ITPM), Siberian Branch, Academy of Sciences of the USSR (1983).
5. A. D. Kosinov and A. A. Maslov, "Development of artificially excited disturbances in supersonic boundary layers," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5 (1984).
6. G. I. Bagaev, V. A. Lebiga, V. G. Pridanov, and V. V. Chernyi, "Supersonic low-turbulence wind-tunnel T-325," in: Aerophysical Investigations [in Russian], Inst. of Theoretical and Applied Mechanics (ITPM), Siberian Branch, Academy of Sciences of the USSR, Novosibirsk (1972).
7. A. G. Elfimov and V. A. Lebiga, Constant-Current Hot-Wire Anemometer [in Russian], Byull. Izobret., No. 24 (1983).
8. J. Max, Methods and Techniques for the Analysis of Signals in Physical Measurements [in Russian], Mir, Moscow (1983).
9. F. J. Harris, "Use of windows in harmonic analysis with discrete Fourier transform," Tr. Inst. Inzh. Elektrotekh. i Radioelektronike, 66, No. 1 (1978).
10. G. Dech, Guide for Practical Applications of Laplace and Z-Transforms [in Russian], Nauka, Moscow (1971).
11. A. D. Kosinov, A. A. Maslov, and S. G. Shevel'kov, Spatial Growth of Wave Packets in Supersonic Boundary Layer [in Russian], Preprint No. 17-85, Inst. of Theoretical and Applied Mechanics, Siberian Branch, Academy of Sciences of the USSR (1985).
12. J. M. Kendall, "Supersonic boundary-layer stability experiments," ARTR-0158 (S3816-63)-1, 2, Sec. 10 (1967).
13. J. Laufer and T. Vrebalovich, "Stability and transition of a laminar boundary layer on an insulate flat plate," J. Fluid Mech. (1960), p. 257.